Topic Study Group 6

Teaching and Learning of Algebra at Primary Level

Jodie Hunter[1], Doris Jeannotte[2], Eric Knuth[3], Ann Gervasoni[4], and Xiaoyan Zhao[5]

1. Aims of the TSG

TSG-6 had a key aim of bringing together a variety of mathematics educators with research interests in early algebra (working with students up to 12 years old). The key overall focus of the group was examining the characteristics and nature of algebraic teaching and learning. This included but was not restricted to the study of numbers, operations, and properties in the context of early algebra, reasoning about functional relationships, the study of structure, and processes related to early algebra such as making conjectures, justifying, generalizing, and developing age-appropriate forms of proof. We called for submissions that offered empirical and/or theoretical contributions and also examined the teaching and learning of early algebra across a wide range of contexts including teacher education, different cultural settings, and with different groups of students.

During the presentations for TSG, we worked collectively to explore themes as well as similarities and differences in relation to the methodologies, theories, and results of both the empirical and theoretical contributions. Overall, we attended to four inter-related areas which included 1) Characteristics and nature of algebraic thinking and reasoning including across different mathematical strands and contexts; 2) Classroom culture and the role of the teacher (including pedagogical practices) in fostering early algebraic thinking for all students; 3) Nature of teacher education and professional development that supports teachers' capacity to foster early algebraic reasoning in classrooms and; 4) analysis of forms of curricular activity that support early algebraic reasoning.

1.1. Submissions

We received 27 submissions from 14 countries (South America: 1; North America: 9; Asia: 6; Middle East: 1; Europe: 6; Australasia: 4) which represented a diverse cultural spread from different international settings. We accepted twenty-three as paper presentations (including both long and short paper presentations) and two as poster presentations, and one submission was rejected. This included three invited paper presentations.

[1] Massey University, New Zealand. E-mail : J.Hunter1@massey.ac.nz
[2] Université du Québec à Montréal, Canada. E-mail : Jeannotte.doris@uqam.ca
[3] The University of Texas at Austin, USA. E-mail: eric.knuth@austin.utexas.edu
[4] Monash University, Australia. E-mail: Ann.Gervasoni@monash.edu
[5] Utrecht University, Netherlands. E-mail: xiaoyanzhao@njnu.edu.cn

1.2. Sessions

TSG-6 hosted three sessions for presentations which were led and moderated by different members of the TSG organisers given the time difference in the respective locations. Each session followed a similar structure beginning with an introduction from the TSG convening group chair and short oral presentations (10 minutes) with questions, answers, and discussion and then long presentations (15 minutes) or an invited presentation (25 minutes) with questions, answers, and discussion. Over the three sessions, we engaged all participants in a short collective discussion at the end of the session to foster networking opportunities along with identifying themes across the research field and to reflect on potential new ideas or existing gaps in the research field.

1.3. Paper topics

Of the 23 accepted papers, only 15 papers were able to be presented during the online conference. A list of these papers and authors are included in order of presentation and are organized in Tab. 1.

Tab. 1. List of papers presented

Papers and author(s)	
[1]	Mathematical learning disabilities in algebra. ***Francesca Gregorio*** (Switzerland).
[2]	The pedagogical journey from arithmetic to algebraic reasoning in a professional development project through the theme of fractions. ***Yuriko Yamamoto Baldin*** *and Aparecida Francisco de Silva* (Brazil).
[3]	Generalizing about odd and even numbers. ***Susanne Strachota***, ***Karisma Morton***, *Ranza Veltri Torres, Ana Stephens, Yewon Sung, Angela Murphy Gardiner, Maria Blanton, Rena Stroud, and Eric Knuth* (USA).
[4]	Toward a common view of algebraic thinking through design of resources by primary and secondary teachers. ***Jana Trgalová***, *Mohammad Dames Alturkmani, and Sophie Roubin* (France).
[5]	Cognitive routes of algebraic thinking in pre-school and elementary school: Literature review. ***Passaro Valeriane***, ***Elena Polotskaia***, *and Azadeh Javaherpour* (Canada).
[6]	Highlighting the potential for developing early algebraic thinking: A praxeological framework of reference. ***Doris Jeannotte***, *Hassane Squalli and Virginie Robert* (Canada).
[7]	Development and implementation of the unit of pattern and correspondence to foster functional thinking. ***Jeongsuk Pang*** (invited speaker) *and Sunwoo Jin* (South Korea).
[8]	The relation between the evolution of generalization and the development of relational thinking and functional thinking: a study with grade 4 students. ***Celia Maria Mestre*** (Portugal).
[9]	Enhancing elementary teachers' functional thinking. ***Ahmad Reza Haghighi*** *and Nasim Asghary* (Iran).
[10]	Arithmetic problems with natural numbers in a multi-grade primary school. ***Lorena Trejo Guerrero*** (Mexico).
[11]	Investigating early algebraic thinking in primary school: An empirical study from China. ***Siyu Sun*** (China).
[12]	Multiplication and division problems as a context for developing young children's algebraic thinking. ***Ann Gervasoni*** *and* ***Anne Roche*** (Australia).
[13]	Young students noticing and generalising growing pattern tasks. ***Jodie Louise Miller*** *and Jodie Hunter* (Australia).
[14]	Designing an evidence-based learning progression for algebraic reasoning. ***Lorraine Day***, *Max Stephens, Marj Horne, and Derek Hurrell* (Canada).
[15]	Fraction tasks which identify algebraic reasoning. ***Catherine Anne Pearn***, *Max Stephens, and Robyn Pierce* (Australia).

In addition, we had two posters presented during the online conference. A list of the posters and authors are included in Tab. 2.

Tab. 2. List of posters presented

Poster and author
[16] Reasoning with patterning tasks. ***Adam Ross Scharfenberger*** (USA).
[17] Research to improve education guidelines for promoting children's understanding of mathematical functions. ***Yoshiki Nisawa*** (Japan).

2. Conference Themes

A review of the conference papers and presentations highlights a range of key themes which were evidence. The first theme is encompassed by the characteristics and nature of algebraic thinking and reasoning including across different mathematical strands and contexts. Passaro and colleagues[5] reported on an analytical literature review to distinguish the approaches identified in research studies to facilitate algebraic reasoning in elementary classrooms. They identified three key approaches across the literature. First, an emphasis on grounding the teaching of algebra in the arithmetic knowledge that students develop. Second, developing tasks to introduce algebraic topics not covered by arithmetic. Third, developing student understanding of quantitative relationships and general laws.

A growing area of interest is the opportunities for early algebraic reasoning across different strands of mathematics. This was represented by a paper which focused on the teaching and learning of algebra through fractions. Pearn and colleagues[15] investigated students' responses to three reverse fraction tasks and from this developed a classification scheme for students' written responses. They argue that students' generalizations related to the strategies that students used to solve the task. Similarly, Guerrero[10] examined how arithmetic problems support students to engage in argumentation and develop understandings of natural numbers. Another paper by Sun[11] highlighted the capability of students from different grade levels in China in solving algebraic tasks. All students demonstrated strengths in in generalized arithmetic tasks, however, older students were more likely to use symbolic representation.

The second key theme identified across the conference papers was the relationship between task design and differing types of algebraic reasoning. Within this theme, there continues to be ongoing interest in the types of reasoning and generalization that students use when asked to solve patterning tasks. Two different presentations, one a poster by Scharfenberger[16] and the second a paper by Mestre[8] illustrate the importance of considering task design and characteristics when supporting students to engage in early algebraic reasoning. Scharfenberger developed a clinical interview using TIMMS items to analyse student responses to patterning tasks. He highlighted that dependent on the context or representation of the pattern, students were provided with differing opportunities to engage in recursive, covariation, and correspondence thinking. Mestre also focused on generalization and used data from a teaching

experiment to analyze the relation between the development of generalization and the development of relational thinking and functional thinking. This study found that task characteristics had the potential for both acting as an enabling factor or barrier to the development of algebraic reasoning. Specifically, pattern exploration tasks and those that focused on relational thinking contributed to functional thinking development.

Also drawing on the theme of task design, a number of papers focused on both the task characteristics and the use of materials to develop algebraic reasoning. Baldin and de Silva[2] focused on using fractions and concrete material to develop algebraic reasoning. These researchers highlight the value of manipulatives in supporting student understanding of operations (multiplication and division) with fractions. This has some overlap, with the work by Gervasoni and Roche[12] who examined multiplication and division problems as a context for developing young students' early algebra understandings. They highlight the important role of models to support students to shift to seeing structure when solving problems. Additionally, Strachota and colleagues[3] examine both task design and identify the affordances of specific tools (concrete material) to support students to make generalizations about odd and even numbers.

Building further on the theme of task design was the facilitation of professional development to support teachers to design tasks and resources. Two papers focused on different aspects of professional development. Firstly, Haghighi and Asghary[9] reported on an intervention that supported teacher capacity to develop and implement resources to support functional thinking in the classroom. Secondly, Trgalová and colleagues[4] worked collaboratively with primary and secondary teachers to design, implement and re-design tasks. Both studies highlight how professional development can support teachers to develop a shared view of algebraic reasoning and the types of classroom activity that support this.

Another key theme was focused on frameworks of learning progressions and the associated teaching approaches to develop algebraic reasoning. Jeannotte et al.[6] highlights the approach of some countries which do not explicitly introduce the development of algebraic thinking in primary school. Their research instead introduces a framework to be used to analyse the potential for developing algebraic thinking in curricula such as textbooks. In contrast, Day and her colleagues[14] report on the explicit development of an evidence-based learning progression across different aspects of early algebra. This included teaching advice designed to support teachers to use a targeted teaching approach to move students along the progression. Three of the papers focused on functional thinking as a route to foster early algebraic thinking. For example, Pang and Jin[7] highlighted the development of a pattern and correspondence unit in a Korean elementary mathematics textbook and the resulting student thinking. Miller and Hunter[13] examined how young students notice structure in growing patterns and the teaching actions that supported this and Nisawa[17] developed a learning framework to support students' understanding of mathematical functions both with the use of numerical values and in other activities without numerical values.

Finally, a key aspect of curricular activity that supports early algebraic reasoning is providing access to all students to engage in early algebra. Gregorio[1] highlights the

lack of equity for specific groups of students including those with mathematical learning difficulties. This research focused on classification of different types of algebra tasks and the development of an assessment tool to identify students with difficulties in accessing algebraic concepts.

3. Areas for Future Research

Across the conference paper and presentations, there continues to be potential for further research to address existing gaps in the field. Firstly, future research could include a focus on diverse and marginalized learners including students from different cultural backgrounds or students with specific learning needs including learning difficulties. Also of interest is a focus on specific teacher actions beyond those related to task design and enactment. This could include questioning and prompts, specific classroom practice, and formative assessment methods that facilitate or support student development of early algebra.

We note that at the conference, there were a number of areas that were under-represented. This includes the use of digital tools such as virtual manipulatives in developing early algebraic thinking. Additionally, while a number of papers focused on task design, learning trajectories or curriculum development, there is a lack of work that focuses on cross-national comparative analyses. Finally, an important addition to the research field would be longitudinal studies which analyse the impact of early algebraic thinking on students' later study of algebra or mathematics in general.